

# Design of Permutation-Based Sparse Code Multiple Access System

Michel Kulhandjian  
School of Electrical Engineering and  
Computer Science  
University of Ottawa  
Ottawa, Ontario, K1N 6N5  
Email: mkk6@buffalo.edu

Claude D'Amours  
School of Electrical Engineering and  
Computer Science  
University of Ottawa  
Ottawa, Ontario, K1N 6N5  
Email: cdamours@uottawa.ca

**Abstract**—We consider the problem of designing a sparse code multiple access (SCMA) codebook that is based on permutation. Conventional SCMA is a multidimensional codebook-based non-orthogonal spreading technique, in which each layer of incoming bits are directly mapped to multidimensional codewords which have nonzero entries on the same resources. In this paper, unlike the traditional SCMA scheme, the data bits to be transmitted determine the nonzero entry locations based on the permutation. The proposed SCMA codebooks are converted to a multiuser code-division multiple-access (CDMA) detection system before applying an iterative decoder. The complexity of the proposed iterative decoder is lower in comparison to exponential message passing algorithm (MPA) and outperforms in terms of bit-error-rate (BER).

## I. INTRODUCTION

There has been considerable attention drawn from the academic and industrial communities in designing the fifth-generation (5G) wireless communication system. To address the demand of high spectral efficiency and massive connectivity it is necessary to have an efficient multiple access technique that requires a shift from orthogonal to non-orthogonal multiple access based approaches [1].

Multiple access is one of the core physical technologies in wireless communication. Most communication systems use orthogonal multiple access, which is achieved by orthogonally dividing resources in one or more dimensions such as frequency, time, code and space. Orthogonality enables easily distinguishable user transmission. However, the drawback of such a system is that its capacity is limited to the number of orthogonal resources available. Increasing beyond this will affect the overall system performance. Non-orthogonal multiple access schemes, which are considered as a key technology for future generation.

From a theoretical point of view, non-orthogonal multiple access expands the capacity region. A well known example of non-orthogonal multiple access is code-division multiple-access (CDMA), which is widely studied and used in practice. The optimal multiuser

detection in CDMA systems is of high complexity, which increases exponentially with the number of users. Low-complexity receivers have been proposed for overloaded systems, which occurs when the number of users is larger than the number of chips per bit, based on the specific structure and decodability properties of the spreading codes [2]-[5]. Although those receivers are linear in the number of users, for larger systems where the overload factor is much greater, complexity needs to be addressed in order for the receivers to be practical.

Low-density spreading CDMA (LDS-CDMA) was introduced in [6] and [7] as a special approach of CDMA for overloaded systems which satisfies the demand of massive connectivity in 5G. In the LDS-CDMA system, modulated symbols are spread over only the nonzero part of spreading codes which are in the domain of  $\pm 1$  and 0. The number of interfering users on each chip is much lower than traditional CDMA. Similar to low-density parity-check (LDPC) code, LDS spreading codes can be represented by a factor graph [8], with variable nodes (VNs) representing data symbols and function nodes (FNs) representing chips. Because of the sparsity of spreading codes, the message passing algorithm (MPA) can be applied for multiuser detection, which has much lower complexity than the optimal maximum a posteriori (MAP) detector.

Recently, LDS has been further extended to sparse code multiple access (SCMA). In the SCMA system, the QAM mapper and the symbol spreader are combined to directly map incoming data streams to multidimensional complex codewords selected from a codebook set. Similarly, SCMA codewords are sparse, which can be represented by a sparse factor graph. Consequently, SCMA can obtain better BER performance than LDS with similar decoding complexity due to its shaping gain, by designing the factor graph and mapping functions.

Unfortunately, designing an optimal SCMA codebook is still an open problem. Hence, suboptimal methods are used for the SCMA codebook design [9], [10]. Although there has been lot of work in literature on

the constellation design [13]-[16] designing a multidimensional constellation with good shaping gains and coding gains for SCMA codebooks is a challenging problem. In [17], the authors design SCMA mapping based on irregular SCMA construction which can handle massive connections and short time delay in the same system simultaneously by assigning different codebooks with various dimensions for different users who demand different performance requirements such as coverage, connections and capacity.

In this work, we consider the problem of designing SCMA codebooks that are based on permutation. Unlike the case of conventional SCMA where incoming bit streams are mapped to complex multidimensional codewords where nonzero resources are fixed for each user here the choice of mapping is determined by the message bits. This irregular construction introduces sparseness which can vary depending on the message bits of each user, therefore it cannot make use of MPA algorithm. However, the advantage of the proposed scheme is that it introduces diversity into the mapping, which is dependent on the message bits. A similar idea was originally proposed for CDMA in [18], for MIMO-CDMA [19] in the presence of frequency selective fading channel and recently adapted for MIMO-OFDM [20] schemes to improve the BER performance in the existence of frequency-selective fading channel with lower complexity. For our proposed permutation-based construction scheme we develop a lower-complexity decoder by first formulating the proposed system model as an overloaded multiuser CDMA system. Based on this formulation we then apply the iterative decoding algorithm [3]. Performance of such a decoding method demonstrates near to ML in terms of BER which is reasonably comparable to MPA algorithm.

The rest of the paper is organized as follows. In Section II we present construction of the conventional SCMA code sets followed by permutation-based in Section III. In Section IV, the iterative low-complexity decoder is presented for proposed permutation-based SCMA scheme. After illustrating simulation results in Section V, a few conclusions are drawn in Section VI.

The following notations are used in this paper. All boldface lower case letters indicate column vectors and upper case letters indicate matrices,  $()^T$  denotes transpose operation,  $\mathbb{C}$  denotes the set of all complex numbers,  $\mathbf{1}$  denotes all ones column vector and  $|\cdot|$ , denotes complex amplitude.

## II. SYSTEM MODEL

Consider SCMA systems with  $J$  users transmitting to a base station. The conventional SCMA encoder, for user  $j \in \{1, \dots, J\}$ , is defined as

$$f_j : \mathbb{B}^{\log_2 M} \rightarrow \mathcal{X}_j, \quad \mathbf{x}_j = f_j(\mathbf{b}_j), \quad (1)$$

where  $\mathbf{b}_j$  incoming bits are first encoded into an  $N$ -dimensional complex vector  $\mathbf{s}_j \in \mathbb{C}^N$ , which is then mapped to  $K$ -dimensional codewords  $\mathbf{x}_j \in \mathcal{X}_j \subset \mathbb{C}^K$  for  $1 \leq j \leq J$  with cardinality  $|\mathcal{X}_j| = M$ . The overload factor is defined by  $J/K$ , whose value is always greater than one. The resulting  $\mathbf{x}_j$  is a sparse complex codeword with  $N < K$  nonzero entries and all the codewords in the codebook contain zero in the same  $K - N$  entries. This operation is defined in the function of  $f_j(\mathbf{b}_j)$ . The SCMA codewords can be represented by a factor graph which contains  $V$  VNs to represent data layers, and  $N$  FNs to represent resources shared by data layers. The edges between VNs and FNs mean the corresponding data layers have nonzero codeword on the associated resources. Fig. 1 illustrates a factor graph that has six VNs which is equivalent to the number of users  $J = 6$ . Each user VN has two neighbor FNs which means  $N = 2$  elements of the codewords are nonzero. Total number of resources, FNs correspond to resources or codeword length of  $K = 4$ .

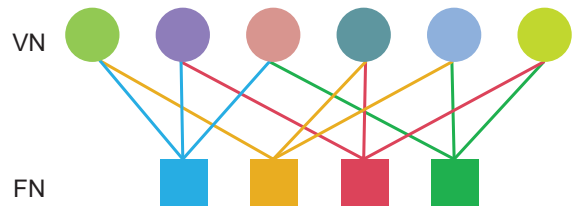


Fig. 1. Factor graph of an SCMA system with  $J = 6$ ,  $K = 4$ ,  $N = 2$

Similarly, we can also represent the same factor graph in a matrix form as in (2). Columns and rows represent the number of users ( $J = 6$ ) and resources ( $K = 4$ ). Factor graph matrix  $\mathbf{F}$  can be expressed in matrix form  $\mathbf{F} = (\mathbf{f}_1, \mathbf{f}_2, \dots, \mathbf{f}_J)$ . User  $j$  and resource  $k$  are connected if and only if  $\mathbf{F}(k, j) = 1$ . An example of factor graph matrix  $\mathbf{F}_{4,6}$  is shown as

$$\mathbf{F}_{4,6} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}. \quad (2)$$

For each user  $j$  encoded complex  $\mathbf{s}_j$  vector is mapped to the locations of ones in corresponding columns in matrix  $\mathbf{F}$ . The resulting  $\mathbf{x}_j \in \mathbb{C}^4$  is sparse and zero locations are the same as in  $\mathbf{f}_j$ . The number of codewords for each user is dependent on the transmitted bit vector  $\mathbf{b}_j$ . If we consider 2-bits information (00, 01, 10, 11) to be encoded into  $\mathbf{s}_{ji} \in \mathbb{C}^2$  complex vectors for  $1 \leq i \leq M$  then  $M = 4$  is the total number of complex vectors of user  $j$ .

### III. PERMUTATION-BASED SCMA DESIGN

In the permutation-based method, unlike the conventional SCMA schemes the encoded  $s_j$  vector is not mapped according to the  $f_j$  column for each user  $j$ . The mapping depends on the incoming information bit stream similar to the idea presented in [19]. Different non-zero locations of encoded complex vectors are assigned to different codewords, hence the technique is referred to as permutation-based SCMA. For this scheme to ensure good performance, large minimum Euclidean distance of a multidimensional constellation is needed with smaller number of collisions between users over a resource. We begin by looking at the following example to demonstrate the motivation. Consider the design of a construction which maps information bits (00, 01, 10, 11) into  $K = 4$  dimensional complex codewords with cardinality  $M = 4$ . To maintain a good minimum Euclidean property within codewords of user  $j$  we seek to maximize the constellation distances when they collide at each  $k$  resource. As an example, for  $J = 6$  users the codebook of users  $j = 1, 2, 3$  can be formed as follows

$$\mathbf{X}_1 = \begin{bmatrix} -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}, \quad (3)$$

$$\mathbf{X}_2 = \begin{bmatrix} 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix}, \quad (4)$$

$$\mathbf{X}_3 = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix}, \quad (5)$$

where each columns represent (00), (01), (10), (11), message bits. It is clear that the minimum Euclidean distance between codewords pairs corresponding to (00) and (11), or (01) and (10) which have maximum Hamming distance is maximized for encoded sparse vectors  $s_j \in \{0, \pm 1\}^4$ . To mitigate the multiuser affect at each  $k$  resource the codebooks of users  $j = 4, 5, 6$  can be constructed by rotating the codebooks of users  $j = 1, 2, 3$  respectively by 90-degree in the complex plane as follows  $\mathbf{X}_4 = \mathbf{X}_1 e^{j\theta}$ ,  $\mathbf{X}_5 = \mathbf{X}_2 e^{j\theta}$ , and  $\mathbf{X}_6 = \mathbf{X}_3 e^{j\theta}$ , where  $\theta = \pi/2$ . In comparison to conventional SCMA codebooks the nonzero entries of permutation-based of each user  $j$  are not at the same resources, as shown in Fig. 2.

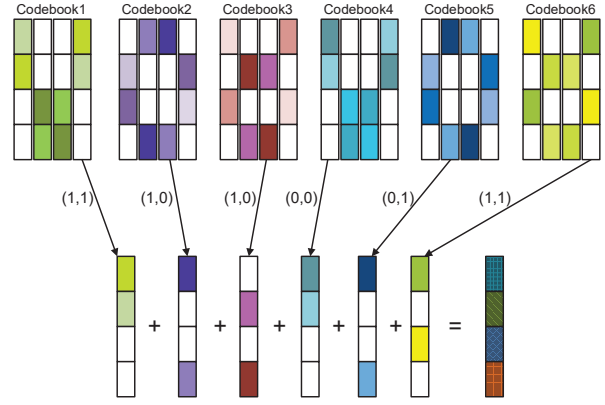


Fig. 2. Permutation-based SCMA codebooks, encoding, and multiplexing.

To extend the above codebook design for size  $M = 16$  we propose to combine two codebooks having size of  $M = 4$ . Corresponding second codewords can be designed by rotating codebook  $\mathbf{X}_i$  of the above example of each users by 45-degree in complex plane  $\bar{\mathbf{X}}_i = \mathbf{X}_i e^{j\theta}$  where  $\theta = \pi/4$  for  $1 \leq i \leq 6$ . Instead of mapping 4-bits information to a codeword from a size of  $M = 16$  we take first 2-bits information to map a codeword from  $\mathbf{X}_i$  and last 2-bits information to map a codeword from  $\bar{\mathbf{X}}_i$ . Those two codewords are combined before transmission. Alternatively if we look at the MIMO system the permutation-based SCMA proposed in this section can transmit  $N_t$  codewords simultaneously on the same  $K$ -resources. This technique has the advantage of increasing the spectral efficiency of the conventional SCMA without increasing the bandwidth or going to higher modulation order. In the case where  $M = 16$  for conventional system we can let in permutation-based system each user to transmit on two antennas,  $N_t = 2$ , using different codebooks  $\mathbf{X}_i$  and  $\bar{\mathbf{X}}_i$ . A similar approach can be applied to the permutation-based SCMA design for the cases of  $M = 64, 256, 1024, \dots$  with  $\mathbf{X}_i^l = \mathbf{X}_i e^{j\theta_l}$  the rotation angles  $\theta_l$  for  $1 \leq l \leq \log_2(M)/2 - 1$  will be the design parameters for the system. For the cases of  $K > 4$  the number of distinct  $f_i$ 's can be produced is  $\binom{K}{N}$  with the assumption of  $N = K/2$ . Take the distinct opposite pairs such that  $f_{j_1} + f_{j_2} = \mathbf{1}$  for each user  $j$  and construct codebook of size  $M = 4$  (e.g.,  $[f_{j_1} \bar{f}_{j_2} - \bar{f}_{j_2} - f_{j_1}]$ , where  $\bar{f}_{j_2}$  is formed by taking alternative ones of  $f_{j_2}$  and negating them). In the case of different codebook sizes  $M$  follow technique explained above.

### IV. MULTIUSER DETECTION

In this section, a low-complexity iterative receiver is proposed based on multiuser overloaded CDMA detection scheme. We begin similar to the conventional the permutation-based SCMA codewords of different users

are multiplexed over shared orthogonal resources, e.g. OFDMA tones. The received signal can be expressed as

$$\mathbf{r} = \sum_{j=1}^J \text{diag}(\mathbf{h}_j) \mathbf{x}_j + \mathbf{n}, \quad (6)$$

where  $\mathbf{h}_j \in \mathbb{C}^K$ ,  $\mathbf{x}_j \in \mathbb{C}^K$  are complex channel coefficients and complex domain codewords selected from corresponding codebooks based on their incoming messages, and  $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I})$  is complex Gaussian noise. Mathematically, we can rewrite the received vector  $\mathbf{r}$  as a multiuser overloaded CDMA system. The proposed codewords  $\mathbf{X}_i$  for each  $1 \leq i \leq 6$  in Section III can be generated by  $\mathbf{C}_i$  matrices.

$$\mathbf{C}_1 = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad \mathbf{C}_2 = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 1 \\ -1 & 1 \end{bmatrix},$$

$$\mathbf{C}_3 = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

As an example,  $\mathbf{X}_1$  can be generated by  $\mathbf{C}_1 \mathbf{z}_1$  for  $\forall \mathbf{z}_1 \in \{\pm 1\}^2$ . Similarly we can form  $\mathbf{C}_4 = \mathbf{C}_1 e^{j\theta}$ ,  $\mathbf{C}_5 = \mathbf{C}_2 e^{j\theta}$ , and  $\mathbf{C}_6 = \mathbf{C}_3 e^{j\theta}$ , where  $e^{j\theta} = \pi/2$ . It is possible now to reformulate (6) using  $\mathbf{C}_i$ 's

$$\mathbf{r} = \sum_{j=1}^J \text{diag}(\mathbf{h}_j) \mathbf{C}_j \mathbf{z}_j + \mathbf{n} \quad (7)$$

$$= \bar{\mathbf{C}} \mathbf{z} + \mathbf{n}, \quad (8)$$

where  $\bar{\mathbf{C}} = [\text{diag}(\mathbf{h}_1) \mathbf{C}_1, \text{diag}(\mathbf{h}_2) \mathbf{C}_2, \dots, \text{diag}(\mathbf{h}_6) \mathbf{C}_6]$  and  $\mathbf{z} \in \{\pm 1\}^{2J}$  is the  $2J$ -dimensional binary information vector. Our objective is the following, given the received signal  $\mathbf{r}$  and  $\bar{\mathbf{C}}$  at the receiver, recover all  $1 \leq j \leq J$  users' codewords. This is equivalent to detecting of binary vector  $\mathbf{z}$  in (8) which is solving multiuser overloaded CDMA detection problem. Generally *under-* or *fully-*loaded scenarios where  $K \leq 2J$  the usual approach is to multiply received  $\mathbf{r}$  by the pseudo-inverse  $\bar{\mathbf{C}}^\dagger$ . The problem is easily solved if all the columns of  $\bar{\mathbf{C}}$  are orthogonal. If they are non-orthogonal columns the problem is exponentially complex because the Maximum Likelihood (ML) detector requires comparison with all the  $2^{2J}$  possible binary vectors. Various sub-optimal solutions can be applied such as sphere-decoding (SD), probabilistic data association (PDA) [3], decision-feedback methods etc.

The problem however becomes more difficult if  $K > 2J$  (*overloaded* case). The complexity of the decoding process is crucial. In [4] Kapur and Varanasi claim that in general linear detectors (e.g. match filter (MF), MMSE decision-feedback receiver) cannot separate users even in the case of asymptotically vanishing noise. Therefore we

seek to change users' gains such that they can be linearly separable to achieve asymptotically efficient multiuser detection when the signal-to-noise (SNR) ratio becomes arbitrarily large. To obtain the gain design algorithm we begin by transforming (8) into the real-valued vector equation

$$\mathbf{y} = \mathbf{G} \mathbf{z} + \mathbf{v}, \quad (9)$$

where

$$\mathbf{y} = [\Re\{\mathbf{r}^T\} \ \Im\{\mathbf{r}^T\}]^T, \quad (10)$$

$$\mathbf{v} = [\Re\{\mathbf{n}^T\} \ \Im\{\mathbf{n}^T\}]^T, \quad (11)$$

$$\mathbf{G} = \begin{bmatrix} \Re\{\bar{\mathbf{C}}\} \\ \Im\{\bar{\mathbf{C}}\} \end{bmatrix}. \quad (12)$$

Note that detection problem in (8) is exactly equivalent to (9). We focus on finding user's gains represented by a diagonal matrix  $\mathbf{A} = \text{diag}(\mathbf{a})$  such that the overloaded multiuser CDMA system is linearly separable. Let us re-write the linear equation with the gains,

$$\mathbf{y} = \mathbf{G} \mathbf{A} \mathbf{z} + \mathbf{v}. \quad (13)$$

Note that we choose  $\mathbf{a}$  in order to keep the total transmit power of the system unaltered (e.g.,  $\mathbf{a}^T \mathbf{a} = K$ ). The iterative procedure for the user's gain design algorithm is summarized in Table I.

TABLE I  
GAIN DESIGN

<b>Initialize:</b> Set $\mathcal{G}(0) = \mathcal{U}$
<b>Step 1:</b> Choose $i_1$ and set $\mathcal{G}(1) = \mathcal{G}(0) - \{i_1\}$ , set $a_{i_1}$ as
$a_{i_1} = \sigma \sqrt{\frac{\Delta}{\mathbf{g}_{i_1}^T \mathbf{g}_{i_1}}}$
<b>Step 2:</b> Choose $i_2$ and set $\mathcal{G}(2) = \mathcal{G}(1) - \{i_2\}$
find $a_{i_2}$ such that $a_{i_2} (\mathbf{g}_{i_2}^T \Omega_{\mathcal{G}(2) \setminus i_2}^{-1} \mathbf{g}_{i_2}) =$
$a_{i_2} \sum_{j \in \mathcal{U} - \mathcal{G}(2) + \{i_2\}} a_j  \mathbf{g}_{i_2}^T \Omega_{\mathcal{G}(2) \setminus i_2}^{-1} \mathbf{g}_j  + \Delta$
...
<b>Step n:</b> Choose $i_n$ and set $\mathcal{G}(n) = \mathcal{G}(n-1) - \{i_n\}$
find $a_{i_n}$ such that $a_{i_n} (\mathbf{g}_{i_n}^T \Omega_{\mathcal{G}(n) \setminus i_n}^{-1} \mathbf{g}_{i_n}) =$
$a_{i_n} \sum_{j \in \mathcal{U} - \mathcal{G}(n) + \{i_n\}} a_j  \mathbf{g}_{i_n}^T \Omega_{\mathcal{G}(n) \setminus i_n}^{-1} \mathbf{g}_j  + \Delta$
Iterate until all users are included

where  $\sigma^2$  is a small value and the Gaussian conditional margin  $\Delta$  is our design parameter. In the following we give all the steps of multiuser overloaded CDMA detection algorithm.

TABLE II  
MULTIUSER DETECTION ALGORITHM

<b>Initialize:</b> Set $\mathbf{x}_{LS} = \tilde{\mathbf{G}}^T (\tilde{\mathbf{G}} \tilde{\mathbf{G}}^T)^{-1} \mathbf{y}$
$[\mathbf{V}, \mathbf{D}, \mathbf{U}] = \text{SVD}(\tilde{\mathbf{G}}), \mathcal{I} = \{\emptyset\}$
<b>For</b> $k = 1$ to $k = 2K$
$\alpha_p = (\mathbf{1} - \mathbf{x}_{LS}) / \mathbf{V}(:, k)$
$\alpha_n = (-\mathbf{1} - \mathbf{x}_{LS}) / \mathbf{V}(:, k)$
$B_p = \text{sign}(\mathbf{X}_{LS} + \mathbf{V}(:, k) \alpha_p^T)$
$B_n = \text{sign}(\mathbf{X}_{LS} + \mathbf{V}(:, k) \alpha_n^T)$
$\mathcal{I} = \cup(B_p, B_n)$
<b>End</b>
Perform bit flipping on the set $\mathcal{I}$ .
Return $\mathbf{x}_m$ that achieves the Euclidean minimum distance.

where  $\tilde{\mathbf{G}} = \mathbf{G}\mathbf{A}$  and SVD is singular value decomposition. Recall that this algorithm returns antipodal vector  $\mathbf{x}_m \in \{\pm 1\}^{2J}$ , which needs to be transformed into estimated codeword. The complexity of the proposed algorithm involves *SVD* computation which is  $\mathcal{O}(n^3)$  where  $n$  is the column size of  $\tilde{\mathbf{G}}$  and the bit flipping with vector computation is linear. To compare with MPA algorithm which has complexity of  $\mathcal{O}(M^{d_f})$  where  $d_f$  is number of users occupying a resource. The next section presents a simulation results of the proposed permutation-based SCMA system BER performance that outperforms the conventional SCMA system.

## V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed permutation-based SCMA technique data dependent mapping with conventional SCMA system. We consider uplink transmission of  $J = 6$  users sharing  $K = 4$  resources. Each complex codeword occupies only  $N = 2$  resources. The following assumptions are used in the simulation model:

- 1) The channel is frequency-selective, a slow Rayleigh fading channel and there is no channel induced inter-symbol-interference (ISI).
- 2) The channel gains are slowly varying circularly-symmetric complex Gaussian random variables with 0 mean and variance 1.
- 3) The channel gains on different transmit and receive links are uncorrelated by assuming sufficient antenna separation.
- 4) The channel gains on different frequencies are uncorrelated by assuming that the frequencies assigned to each SCMA symbol are a subset of a much larger OFDM frame and that they are sufficiently separated within this frame to assume them to be uncorrelated.
- 5) We assume perfect channel state information (CSI) is available at the receiver.
- 6) We do not perform any channel coding on codewords.

For comparison purposes we utilize conventional SCMA construction schemes proposed in [10]. The Log-MPA decoder [11] algorithm applied to conventional SCMA to compare the BER performances. We use multiuser detection algorithm presented in Table II to decode the proposed scheme. Both conventional and permutation-based SCMA performance are evaluated by calculating the BER for different energy per bit to noise power spectral density ratios ( $E_b/N_0$ ). In Fig. 3 we illustrate the motivating example of codebook size  $M = 4$  where we plot the BER performance averaged over the different users. We noted that the BER performances of the individual users are similar to this average with minor fluctuations, which fall within the expected confidence interval. The BER performance of permutation-based

SCMA is better than conventional SCMA. Gains for permutation-based SCMA are not computed in this specific example. One might raise question of shaping gains of conventional SCMA is negligible for the spreading factor of  $K = 4$ . Due to space limitation of this paper, we will not demonstrate simulations for various  $J$  and  $K$  instead focus on  $J = 6$  and  $K = 4$  in this paper.

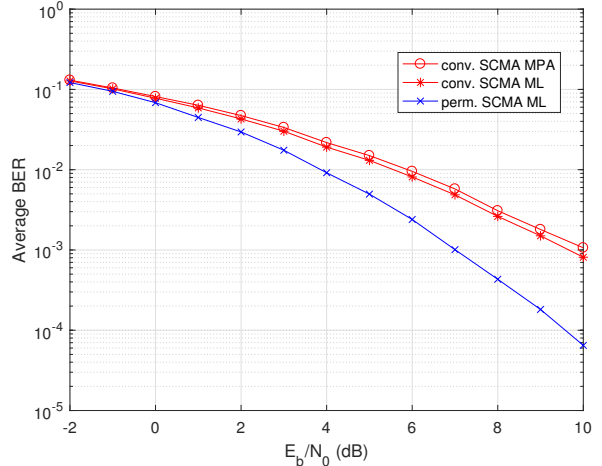


Fig. 3. One transmit antenna  $M = 4$ .

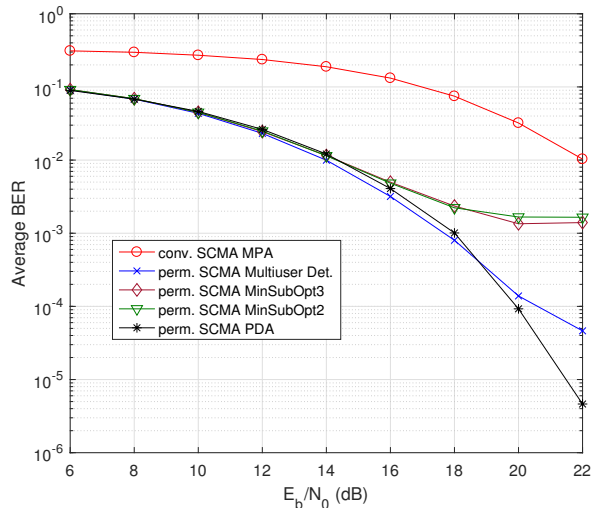


Fig. 4. Two transmit antennas with  $M = 16$  and  $M = 4$ .

For the case when  $M = 16$  instead of single transmit antenna we illustrate performance with two transmit antenna in Fig. 4. The conventional SCMA encoding for each user takes 4 bits of information then converts them into codewords and transmits using two transmit antenna to have transmit beamforming diversity. The proposed scheme, as described in Section III, each user  $j$  takes 2 bits of information to transmit on each antenna that has  $M_t = 4$  number of codewords for  $t \in \{1, 2\}$ . Combined

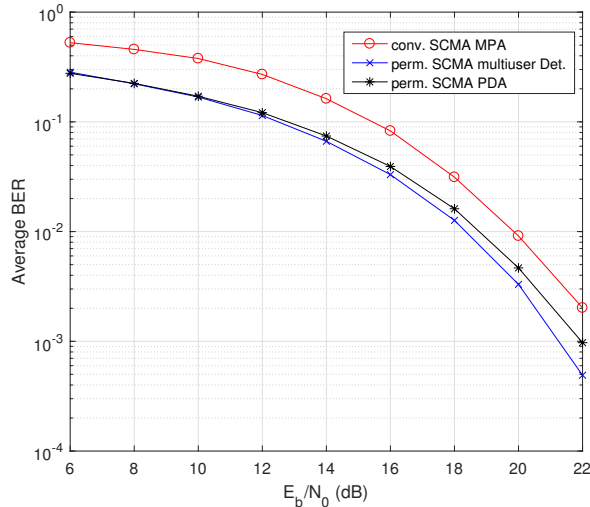


Fig. 5. Two transmit antennas with  $M = 4$ .

information bits from two antennas is 4, which corresponds to total of  $M = 16$  codebook size. The BER performance of permutation-based SCMA is superior to conventional SCMA system. Looking at  $E_b/N_0 = 18$  dB error rates are about  $10^{-3}$  and  $10^{-1}$  for permutation-based and conventional SCMA schemes, respectively. In Fig. 5 we plot BER performance with  $M = 4$  for conventional SCMA using two transmit antenna similar to permutation-based SCMA scheme and applied joint-decoding algorithm presented in [12].

## VI. CONCLUSION

In this paper, we have introduced a new permutation-based SCMA scheme. The proposed permutation-based SCMA system achieves better BER performance compared to conventional SCMA, because of the diversity achieved by assigning different codewords, which depend on the transmitted bits. Moreover, the proposed system provides better spectral efficiency by dividing bits to have smaller codeword sizes then combining, or even better transmitting on multiple antennas, with lower number of constellation points. The computational complexity of the detector of our proposed system is lower in comparison to exponential MPA decoding algorithm.

## REFERENCES

- [1] L. Dai, B. Wang, Y. Yuan, S. Han, C.-L. I, and Z. Wang, "Nonorthogonal multiple access for 5G: Solutions, challenges, opportunities, and future research trends," *IEEE Commun. Mag.*, vol. 53, no. 9, pp. 74-81, Sep. 2015.
- [2] M. Kulhandjian and D. A. Pados, "Uniquely decodable code-division via augmented Sylvester-Hadamard matrices," in *Proc. IEEE (WCNC) 2012*, Paris, France, Apr. 2012, pp. 359-363.
- [3] G. Romano, F. Palmieri and P. K. Willett, "Soft iterative decoding for overloaded CDMA," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP '05)*, Philadelphia, U.S.A., vol. 3, pp. 733-736, March 2005.

- [4] A. Kapur, M. K. Varanasi and C. T. Mullis, "On the limitation of generalized Welch-bound equality signals," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 2220-2224, June 2005.
- [5] A. Singh and P. Singh, "Uniquely decodable codes for overloaded synchronous CDMA with two sets of orthogonal signatures," in *Proc. IEEE Annual India Conference (INDICON) 2014*, Pune, India, pp. 1-4, Dec. 2014.
- [6] R. Hoshyar, F. P. Wathan, and R. Tafazolli, "Novel low-density signature for synchronous CDMA systems over AWGN channel," *IEEE Trans. Signal Process.*, vol. 56, no. 4, pp. 1616-1626, Apr. 2008.
- [7] J. van de Beek and B. M. Popović, "Multiple access with low-density signatures," in *Proc. IEEE GLOBECOM 2009*, Honolulu, Hawaii, USA, pp. 3787-3792, Nov./Dec. 2009.
- [8] F. Kschischang, B. Frey, and H. Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Trans. on Info. Theory*, vol. 47, no. 2, pp. 498-519, Feb. 2001.
- [9] H. Nikopour, and H. Baligh, "Sparse code multiple access," in *Proc. IEEE 24th PIMRC*, pp. 332-336, Sep. 2013.
- [10] M. Taherzadeh, H. Nikopour, A. Bayesteh, and H. Baligh, "SCMA codebook design," in *Proc. IEEE 80th Veh. Technol. Conf. (VTC Fall)*, Vancouver, BC, Canada, pp. 1-5, June 2014.
- [11] J. Liu, G. Wu, S. Li and O. Tirkkonen, "On Fixed-Point Implementation of Log-MPA for SCMA Signals," *IEEE Wireless Communications Letters*, vol. 5, no. 3, pp. 324-327, Jun. 2016.
- [12] Y. Du, B. Dong, Z. Chen, P. Gao, and J. Fang, "Joint Sparse Graph-Detector Design for Downlink MIMO-SCMA Systems," *IEEE Wireless Communications Letters*, vol. 6, no. 1, pp. 14-17, Feb. 2017.
- [13] G. D. Forney, Jr., and L. F. Wei, "Multidimensional Constellations Part I: Introduction, Figures of Merit and Generalized cross," *IEEE Journal on selected areas in communications*, vol. 7, no. 6, pp. 887-892, Aug. 1989.
- [14] M. Beko and R. Dinis, "Designing Good Multi-Dimensional Constellations," *IEEE Wireless Communications Letters*, vol. 1, no. 3, pp. 221-224, Jun. 2012.
- [15] H. Wu, J. Zhang, H. Song, "Lattice Based Approach to the Construction of Multi-Dimensional Signal Constellations," *ACTA Electronica Sinica*, vol. 42, no. 9, pp. 1672-1679, Sep. 2014.
- [16] D. Cai, P. Fan, X. Lei, Y. Liu and D. Chen, "Multi-Dimensional SCMA Codebook Design Based on Constellation Rotation and Interleaving," in *Proc. IEEE 83rd Veh. Technol. Conf. (VTC Spring)*, Nanjing, China, pp. 1-5, May 2016.
- [17] S. Zhang, B. Xiao, K. Xiao, Z. Chen, and B. Xia, "Design and Analysis of Irregular Sparse Code Multiple Access," *IEEE International Conference on Wireless Communications & Signal Processing (WCSP 2015)*, Nanjing, China, pp. 1-5, Oct. 2015.
- [18] C. D'Amours, "Parity Bit Selected Spreading Sequences - A Block Coding Approach to Spread Spectrum," *IEEE Commun. Letters*, vol. 9, no. 1, pp. 16-18, Jan. 2005.
- [19] M. Shi, C. D'Amours, A. Yongacoglu, "Design of Spreading Permutations for MIMO-CDMA Based on Space-Time Block Codes," *IEEE Commun. Letters*, vol. 14, no. 1, pp. 36-38, Jan. 2010.
- [20] S. Moussa, A.O. Dahmane, C. D'Amours and H. Hamam, "MIMO-OFDM scheme based on permutation spreading," in *Proc. 15th Int. Conf. Advanced Communication Technology (ICACT 2012)*, Pyeongchang, Korea, pp. 679-682, Jan. 2012.